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ABSTRACT

The optimal use of representations in furthering mathematical discourse of high quality may depend on the function a representation fulfills during task completion. Tasks may be differentiated into Low Structure Representation (LSR) tasks (open task statement with emergent use of representations), and High Structure Representation (HSR) tasks (explicit use of representations in the task statement). This study identifies differences in the instructional discourse associated with HSR and LSR tasks. Since HSR tasks unnecessarily constrain problem solving, it is hypothesized that instructional discourse will incorporate less negotiation of mathematical meanings than discourse associated with LSR tasks. This study uses a subsample of 60 lessons from the TIMSS-Video study, which was part of the Third International Mathematics and Science Study (TIMSS). It suggests that the two indices for quality of instructional discourse--structure of discourse with regard to the extent of interconnectedness, and content of discourse with regard to its treatment of mathematical procedures and explanation--do not occur independently of each other. Appended are high and Low Structure Representation Tasks. (Contains 21 references and 1 table.) (ASK)

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The Relationship between the Use of Representations and Instructional Discourse in Mathematics Tasks

Introduction

The use of visual representations such as graphs and diagrams is considered essential for the familiarization of students with the thinking of experts in mathematics and science (e.g., Cobb, 1995; McGinn & Roth, 1999). Experts routinely employ representations for communication of knowledge and development of problem-solving methods (Roth & McGinn, 1997; 1998). However, representations should not only be taught in schools because they are part of the repertoire of experts, this embodying the danger of a mere procedural approach to their teaching. More importantly, it is the potential of representations to function as tools for communication and problem-solving, that, if instantiated in instruction, enables students to gain new insights into mathematical structures.

The tool-like use of representations in instructional tasks offers the potential for student activities at high cognitive levels as it supports the integration of solution methods and conceptual connections (e.g., Dreyfus, 1991; Kaput, 1989). These instructionally valuable student activities, however, are often not realized in mathematics classrooms, where the mere description of problem-solving methods without relation to broader concepts prevails (Stein, Grover, & Henningson, 1996). The cognitive demands of student activities intended by a task are not necessarily congruent with the way the task is actually implemented, and especially cognitively challenging tasks frequently are worked on in rather limiting ways (Henningson & Stein, 1997). This low-level implementation is evident in instructional discourse, which tends to focus on procedural issues of task solutions, with little room for negotiation of mathematical meanings. Instructionally useful discourse consists of exchanges in which students discover relationships between quantities and solution methods and extrapolate beyond existing knowledge (e.g., Hiebert, Carpenter, Fennema, Fuson, Human, Murray, Olivier, & Wearne, 1996). In a socio-constructivist view, this high quality instructional discourse and students' construction of understanding co-constitute each other (e.g., Cobb, Gravemeijer, Yackel, McClain, & Whitenack, 1997; Hiebert & Wearne, 1993). Social situations such as group work or teacher-initiated classroom discourse afford students' expressing their reasoning by means of verbal elaboration, description, or questioning. This process of communicating one's conceptualization of a problem to others thus means that common meanings are negotiated turn by turn. Students co-construct their mathematical insights, reflected in a structure of interconnected instructional discourse based on the use of mathematical symbols (see also Sfard, 2000).

Which may be the conditions supporting instructionally relevant mathematical discourse, and what contribution may derive from the use of representations? Mathematical tasks have been proposed as a major influence on student learning by many (e.g., Doyle, 1983, Henningson & Stein, 1997; Renkl, 1991). Depending on the types and cognitive levels of mathematical activities intended by tasks, the nature of classroom discourse will vary. For example, Hiebert and Wearne (1993) found that teacher questions (i.e., tasks; Renkl, 1991) asking students for explanations were associated with classroom discourse covering a higher range of topics and containing longer student contributions than discourse associated with procedural questions. In a broader conceptualization of tasks, however, classroom discourse associated with task implementation will be influenced by more than the task statement. Henningson and Stein (1997) suppose that adequate structural support provided to students allows for an appropriate implementation of a high level mathematical task, whereas too much structural support (e.g., providing hints) may remove challenging aspects of a task and lead to student activities of lower cognitive levels than the task statement would imply.

Representations may provide just this element of structure which is important during task implementation. By explicating structural properties and relationships between quantities, they on the one hand offer a point of reference to the learner, which may serve as an anchor for the (verbal) construction of mathematical meaning. They hereby allow the reconceptualization and integration of problem-solving methods by students. On the other hand, the employment of representations may prevent students from elaborating mathematical content, if their use suggests particular solution procedures and unnecessarily constrains the problem-solving space. The pre-given structure of content invites a mere reproduction of solutions, making unnecessary conceptual connections, such as the need to choose between solution methods.

The optimal use of representations in furthering mathematical discourse of high quality may then depend on the function a representation fulfils during task completion.

That is, whether representations will provide necessary structure, leading to successful task completion, or too much structure, inhibiting high level student activities, may depend on the way they are employed. When tasks do not suggest a particular solution method, the use of representations as structural support may emerge during problem solving. However, when the employment of a representation is stated explicitly, the task may become overly structured. Here, the structure inherent in representations constrains student activities, leading to a procedural focus during task completion. Tasks may thus be differentiated into Low Structure Representation (LSR) tasks (open task statement with emergent use of representations) and High Structure Representation (HSR) tasks (explicit use of representations in the task statement).

The effects of differential use of representations may be especially evident in lessons introducing new mathematical content. Here, the potential of representations for fostering conceptual understanding is great, while their procedural use (which may be justified in practice phases) is especially limiting. Goal of this study is to identify differences in instructional discourse associated with HSR tasks and LSR tasks. Since HSR tasks unnecessarily constrain problem-solving, it is hypothesized that instructional discourse will incorporate less negotiation of mathematical meanings than discourse associated with LSR tasks.

Method

In the TIMS-Video Study, which was part of the Third International Mathematics and Science Study, 231 eighth-grade classrooms from three countries (Germany, Japan, United States) were videotaped for one 45-minute mathematics lesson (Stigler, Gonzales, Kawanaka, Knoll, & Serrano, 1996). A subsample of 60 lessons, with 20 lessons from each country, was randomly chosen. Based on secondary analyses of tasks of introductory phases by Knoll (2001) at the Max-Planck-Institute, Berlin, all tasks intending a student activity of "representing," that is, the translation between symbol systems (verbal, symbolic, and iconic), were determined. A task in this conceptualization refers to all teacher statements intending a mathematical student activity (Renkl, 1991). Tasks may be classified into superordinate and subordinate tasks, with the latter category referring to teacher follow-up questions during task implementation of a superordinate task. The task of "representing" presupposes an understanding of relationships between symbol systems, offering the opportunity for sophisticated mathematical reasoning.

Two categories of representation tasks were formed, differentiating High Structure Representation Tasks (task statement involves the direct translation between symbol systems, suggesting a particular solution procedure, e.g., "Draw a triangle with double length sides," "Determine the equation corresponding to these functions drawn in a coordinate system") and Low Structure Representation Tasks (task statement leaves open choice of symbol system and solution procedure, e.g., "Try to prove the relationship between length of a triangle side and the angle that lies on the opposite side," "Find reasons why the sum total of PQ and PR are the same as AB."). While HSR tasks focus on one property of a representation such as the construction of a slope, LSR tasks leave open which property of a representation may be focused on and how it may be employed to support task solutions.

The quality of task implementation was determined by analysis of instructional discourse occurring after the task statement. Only public talk was considered, i.e., talk during phases of independent or group seat work was excluded from analysis. A task implementation ends with the statement of another task. If the representation task was followed by subordinate task statements (teacher follow-up questions), the next superordinate task statement marked the end of the implementation phase.

Quality of instructional discourse is evident in 1) content of discussion and 2) structural characteristics of discourse. In content, high quality discourse focuses on the connection between symbol systems and solution methods, often involving mathematical explanations (Hiebert et al., 1996; NCTM Standards, 1989). Analyzed with a focus on structural characteristics, high quality instructional discourse will contain indices of negotiation of meaning, as identified by means of discourse analysis (i.e., connected teacher-student exchanges with sequences of argument-counterargument-repair-acceptance or question-reaction-reaction-reaction-acceptance). Content of low quality discourse involves the reiteration of procedures and mathematical facts without connecting them to broader mathematical concepts. Its structure consists of short sequences of question and answer, without the need to repair statements or go beyond content already provided.

Results

Cognitive Demands of Intended Student Activities

Tasks were classified according to their intended student activity and according to the cognitive demands that these intended activities put on students. In a categorization of cognitive demands of task statements, three major categories may be identified (Knoll, 2001; see also Baumert, Bos, & Lehmann, 2000; Bloom et al., 1956). Routine tasks as the lowest category are based on activation and reproduction of known content or procedures in order to identify, judge, or describe a known situation. For example, students may be asked to solve for X in a functional equation. Application tasks intend students to apply known content to a situation which makes necessary a transformation of solution procedures. They thus presuppose a basic understanding of mathematical procedures and how to use them. For example, students may already know how to draw a graph in a coordinate system given the functional equation. The application tasks now intends students to determine the functional equation given a graph in the coordinate system. Problem solving tasks, finally, involve the construction of new approaches to a given problem and presuppose an understanding of the structure of a problem in relation to the use of mathematical methods to arrive at an answer. Here, students need to know how a particular procedure may be adapted or developed for a problem of a certain structure, that is, they need a structural model of the mathematical situation. The use of visual representations thus may be particularly helpful for higher-level cognitive tasks. For example, students may be asked to develop a method to determine the area of a circle with a certain radius, without yet knowing the appropriate formula (Knoll, 2001).

Of the 1549 tasks employed in introductory phases of the 60 mathematics lessons, only 44 (2.8%) involved the intended student activity of representing quantities expressed in one symbol system in terms of another, as for example the translation of an equation into a graph in a coordinate system. The cognitive demands that these representation tasks pose are generally high, with 55% routine tasks, 30% application tasks, and 15% problem-solving tasks. In contrast, the majority of tasks in introductory phases were rated as routine tasks (87%), with only 9% application tasks and 4%

problem-solving tasks (Knoll, 2001; reliabilities for classifications were satisfactory). Tasks based on representation of mathematical content in various symbol systems thus do seem to offer the potential for high level student activity to a higher degree than other tasks.

Of the 44 representation tasks, 12 were classified as LSR and 32 as HSR tasks. Based on this differentiation of representation tasks it becomes evident that it is the LSR tasks that are rated at high cognitive levels, with 42% problem solving tasks, 33% application tasks, and 25% routine tasks. HSR tasks, in contrast, are similar to the ratings of tasks without the use of representations, with 1% problem solving tasks, 32% application tasks, and 67% routine tasks, see Table 1. The use of representations constrained to one explicitly stated procedure in HSR tasks is thus evident in their ratings of cognitive demands as intending routine mathematical procedures, whereas the employment of representations in an open-ended fashion in LSR tasks is reflected in their more frequent ratings as problem solving tasks.

Table 1. Percentages of cognitive levels of different types of tasks.

	Types of tasks		
Cognitive level of task statement	All Introductory Tasks	High Structure Representation Tasks	Low Structure Representation Tasks
Routine	87%	67%	25%
Application	9%	32%	33%
Problem Solving	4%	1%	42%

For a comparison of the instructional discourse associated with HSR tasks and LSR tasks, each five tasks were selected, controlling for cognitive demands of intended student activity. Following the rationale that a difference in quality of instructional discourse due to type of representation task will be most evident in task statements of comparable cognitive demands, tasks posing the cognitive demand of application seemed most appropriate. Importantly, application tasks leave room for task implementations at high and low quality of instructional discourse compared to the task statement, more so than this may be the case for task statements with extremely challenging or simple demands on students. Thus, all of the available five application tasks of the HSR category were compared to all of the four application tasks of the LSR category plus one randomly chosen problem-solving task of the LSR category. A list of the ten task statements and their classifications is provided in Appendix A.

Structure of Instructional Discourse during Task Implementation

In order to characterize the structure of instructional discourse during the implementation of LSR tasks and HSR tasks, the degree of interconnected instructional discourse was assessed. Assessing the degree of interconnected instructional discourse as indication for the quality of task implementation follows the rationale that, in order for students to develop a mathematical argument, the turn-taking structure of discourse should be elaborate, that is, involve sequences of argument and response which are co-constructed by teacher and students. If, however, instructional discourse consists of short sequences, it is likely that arguments are merely presented rather than developed collaboratively or negotiated. Thus, little opportunity for students to build on and move beyond their own conceptualizations arises. Note that, in order for discourse to be classified as a connected sequence, at least two speakers need to contribute. Thus, for example, extensive teacher talk without connection to utterances of students in turn-taking will not be included in an index of connected discourse. For this analysis, discourse codes developed by G. Schümer and others (1997), based on analyses of discourse by Stigler et al. (1996), were employed. For a coding of interconnected discourse sequences, only content-related utterances are considered. That is, teacher statements directed at student management, for example, are excluded since they likely do not contribute to the negotiation of meaning. In interconnected discourse, the shortest meaningful sequence consists of an elicitation, a response, and reaction to the response. An elicitation of different content marks the beginning of a new sequence. Sequences may be embedded in larger sequences, all referring to the same topic. For example, a teacher may collect different student answers to a question, with each student answer occurring in a separate turn-taking sequence. Here, the entire sequence including short sequences would be considered as one connected piece of discourse.

LSR tasks showed a significantly higher degree of interconnected classroom discourse than HSR tasks (Mann-Whitney U $(N=10) = 4$; $p< .05$, one-sided). This is evident in a comparison of the percentages of public instructional discourse that occurred in sequence for the two task types. In LSR tasks, 67% of public talk occurred in connected sequences, whereas in HSR tasks only 39% of public talk was connected. Of this interconnected discourse, LSR tasks showed a significantly longer mean length of sequences than sequences associated with HSR tasks, with $\underline{M}$ (LSR) = 8.9 related utterances and $\underline{M}$ (HSR) = 4.7 related utterances (Mann-Whitney U $(N=10) = 4$; $p < .05$, one-sided). Thus, while discourse associated with HSR tasks typically involved short sequences of teacher question and student answer(s), followed by a teacher reaction, discourse associated with LSR tasks more extensively dealt with the negotiation of one topic, with a turn-taking structure of several related utterances. Generally, short sequences imply a higher dependence on teacher direction due to their typical scheme of question-elicitation-answer-reaction. In longer sequences, students may be given the opportunity to develop their thinking in several steps, supported by comments and leading questions of the teacher. Since in HSR tasks, a majority of public talk did not even include these short sequences of interconnected discourse, it may be assumed that much of the instructional discourse was driven by teacher monologue. In fact, students in HSR tasks contribute only 20% to turns of public discourse.

In the following, the difference between short and extended connected sequences of instructional discourse is illustrated by two cases. The first example involves a short sequence of three related utterances, occurring during implementation of a HSR task which intends students to use information about the slope and intercept of a graph to construct an equation.

Teacher:	how much is the slope?
Student:	negative two
Teacher:	negative two, right? Okay.

After posing a precise question, the student provides the requested information. Since the proposed answer is correct, there is no (perceived) need for further investigation into the process by which the answer was obtained. By posing questions eliciting other pieces of information, the teacher moves on to demonstrate how to translate the equation into a graphical representation.

In contrast, a longer sequence of related utterances, associated with the implementation of a LSR task of figuring out the speed of an airplane with which students could experiment, exemplifies elements of interconnected discourse:

Student:	well, then we figured out the circumference.
Student:	okay?
Teacher:	Okay. S I'm not going to ask you to tell us that calculation. You figured out the circumference.
Student:	Yeah. And then uhm we turned what-- we multiplied the circumference by how many times it's gone around in a minute.
Teacher:	Okay. So you did 73 times whatever the circumference turned out to be.
Student:	which was
Teacher:	don't tell me. Just a brief summary.
Student:	okay. Okay.
	And then we changed it into feet.
Teacher:	okay, because these are inches.
Student:	it was in inches.
Teacher:	okay.
Students:	And then we multiplied that by sixty to get how many times it went around in an hour. And how many times it went around in a minute.
Teacher:	okay. Okay.
Student:	And then we divided that by.. five thousand
Teacher:	two hundred and eighty.
Student:	two hundred and eighty to get how many because that's how many.
Teacher:	okay.

In this sequence, the student presents the group's approach to solving the problem, supported and commented on by the teacher. Frequent positive acknowledgement by the teacher seems to be an important element for keeping the conversation going. Twice, the collaborative completion of a statement by student and teacher together is evidenced, suggesting a common ground of conversation.

In a comparison of the range of values for the five LSR tasks and the five HSR tasks it is evident that in each group the majority of cases are quite homogeneous with respect to the average number of interconnected statements. In both groups, however, there is each one case with an exceptionally high average. In the LSR task group, this means that one case shows an average number of 19.7 connected statements, which is greatly above the group mean of 8.9. In the HSR task group, there is one case with an average number of 11.3 connected statements, which thus is even above the LSR task group mean. The importance of this case for further investigation of the use of representation tasks in instruction will be discussed later.

Although different degrees of interconnected discourse seem to be associated with different types of task statements (i.e., whether the task is a HSR task or a LSR task), there is the possibility that instructional discourse is mainly driven by the instructional style of a teacher. That is, some teachers may naturally create a learning environment of highly interconnected discourse, whereas others may teach in a style resembling the question-answer pattern evidenced in the first transcript. In order to make possible comparisons of degree of interconnected discourse for tasks implemented by the same teacher, all other tasks stated at a level of application were identified for the transcripts under consideration. In four transcripts, application tasks without the use of representations could be matched with representation tasks. Results showed that in each comparative case, the degree of interconnected discourse associated with the implementation of application tasks without representations was lower than for discourse associated with representation tasks. This was the case even for comparisons of HSR tasks (already associated with a low degree of connected discourse) with their respective application task implementations. Thus, some supporting evidence for the impact of representation tasks on instructional discourse beyond personal teaching styles has been collected.

Mathematical Content of Instructional Discourse during Task Implementation

For an analysis of mathematical content of instructional discourse associated with application tasks, one distinction is especially pertinent, referring to the role of mathematical explanations and connections between procedures. Is the solution of the problem executed in a procedural fashion, reiterating the steps to be taken to arrive at the solution? Or does the solution process incorporate explanations of why and how certain steps in problem solving were appropriate? While the first type of task implementation is focused on the correct application of mathematical procedures, the second type involves rationales for and connections between different types of mathematical procedures, relating them to broad mathematical concepts. Task implementations that involves mathematical reasoning and conjecturing of students have been called tasks of "doing mathematics," relating to the active use of mathematical tools (Henningson & Stein, 1997). Procedural task implementations, on the other hand, embody a more mechanistic stance on the nature of mathematics, focusing on the correct execution of solution methods. This type of implementation may be especially frequent for application tasks since here the task statement demands for an application or transformation of procedures to new mathematical content. Thus, a reiteration of how a certain procedure was applied may be afforded especially well by the task statement. However, application tasks may also afford looking into the connection between different solution methods and relating them to the structure of a mathematical problem if, for example, the task statement leaves room for the application of different solution strategies. Thus, although high-level mathematical reasoning may more likely be associated with tasks at the level of problem solving, a particularly open task statement (as in LSR tasks) may leave room for or even demand discussion of the connection between different procedures rather than their mere reiteration.

The implementation phase of each of the ten tasks was rated as to their 1) discussion of procedural steps or 2) statement of connections to other mathematical concepts and/or other procedures. For the HSR tasks, in four out of the five task implementations the goal of establishing a "recipe" of how to do a certain procedure seems to have been pursued. For example, in a task asking students to find the linear equation of a graph drawn at the blackboard by checking the amounts for a and B [in Y = aX + B], the steps of how to find the value for the y-intercept and the value for the slope are presented. After the correct numerical answers are stated by students, the teacher summarizes

> "It's just that if there is an area that clearly passes through whole numbers then you will know right away what the intercept is. So first of all if it can be solved then solve for the intercept right away, okay? Then take steps to solving the slope, is the method that should be taken. Okay?"

Nevertheless, some HSR task implementations do involve either the statement of connections between symbol systems or a relation to broader concepts. If connections are mentioned they are, however, stated solely by the teachers. That is, the main contribution of students to instructional discourse is the statement of procedural steps taken to arrive at the solution. This is evident for example in one case, where the task statements intends students to sketch graphs of quadratic functions in coordinate systems supported by the use of graphing calculators. Students follow a set procedure to arrive at the graphical representations. During discussion of seatwork, students state the value of the discriminant they found for different quadratic functions, depending on their X intercepts. While students read off their numerical solutions, it is the teacher who then connects their statements to the conclusion that the discriminant is "telling us how many real solutions we have." The teacher now refers to a sheet which lists the number of solutions associated with positive, negative, or zero discriminants. Although there may have been the potential for closer examination of the relation between number of x intercepts in a graphical representation and the discriminant, this room for discussion of the relation between symbol systems is not exploited. Overall, there is only one HSR task implementation that establishes connections between different symbol systems--the one HSR task of unusually highly connected instructional discourse mentioned above.

In LSR tasks, three tasks with a clear focus on explanation of mathematical concepts and relations could be determined. For example, one task asks students to construct a square for a binomial formula locating each part of the formula in the square. After appropriately labeling the sides of the square, teacher and students collaboratively build on an explanation of why it may be helpful to use squares with binomial formulas, namely, to visually check the correctness of a formulaic answer. Two LSR tasks, however, pursue a more procedural focus in their implementation phases. As in HSR tasks, some connecting statements by teachers do occur even in procedural task implementations; however, these statements are, in contrast to teacher statements in HSR tasks, also anchored in the use of the respective representations. That is, teachers presents conceptual mathematical knowledge which they relate to the respective elements of a representation, for instance by pointing at the blackboard. It may be hypothesized that a teacher comment which is supported by the use of a visualization may effectively relate to students' own conceptualization of the mathematical problem if the representation has been used as a reasoning tool in the previous part of the conversation. That is, the representation may function as a bridge between students' thinking and the teacher's provided explanation of mathematical concepts.

The Role of Representations during Task Implementation

Analyzing the different ways representations are employed during task implementation, it is evident that representations are used in a more restricted fashion in HSR tasks than in LSR tasks. Task statements of HSR tasks focusing on one aspect of a representation thus seem to be a limiting factor also during implementation. Often, task implementations of HSR tasks involve the one-to-one translation between symbol systems, as for example when a teacher asks specific questions directed at illustrating the translation of information given in a symbolic expression into the construction of a table and then into the drawing of a graph. Although a visual representation is constructed in the process of task implementation, it is not used as a basis for explaining broader concepts such as how the slope of a different equation may compare to the one constructed and how this may be predicted from the equation.

In LSR tasks, representations are frequently used as an tools for mathematical reasoning. Since LSR tasks involve task statements that leave open which element of a representation may be focused on during the solution process, students more frequently need to recur to the representation to explain their reasoning. For example, in a LSR task pertaining to the proof of sides relationships in a triangle ("Why the sum total of PQ and PR are the same as AB. Please find reasons for it," entire task statement see

Appendix A), the visual representation functions as a tool for problem solving and extensive discussion of solution procedures: The diagram with marked sides is provided at the blackboard. Students first solve the task individually, after which follows a demonstration of several student-generated solutions at the blackboard. During the demonstrations, the teacher questions the approaches taken in order to have students explain their reasoning for choosing a particular method. During explanation of solutions, the diagram is used to locate arguments such as the assertion that a rule about equivalence of base angles may be employed.

Student:	(...) and we found out that this angle and this angle are corresponding angles then since the base angles are equal we found out that this is an isosceles triangle.
Teacher:	Where...the ba- base angles are the same. Where and where are the base angles?
Student:	here and here (points at diagram)
Teacher:	oh that's right. Yes, yes, yes, yes.
Student:	Since they are equal we know that it's an isosceles triangle and
Teacher:	yes.
Student:	since this side and this side of an isosceles triangle are equal so
Teacher:	yes
Student:	then the side here and the side here are the same.

During this phase of explanation, the representation functions as an anchor, enabling the visualization of mathematical assumptions made in the course of task solutions. Importantly, its use also enables other students to follow the reasoning of the presenting students both verbally and visually. This becomes clear in the part of instructional discourse following students' presentation of solution methods, which summarizes the importance of the parallelogram in the figure.

Teacher:	A parallelogram. So what was it? This and this. (...) What does it mean? What does it mean? Okay then Shibuya.
Student:	the lengths of the sides facing each other are equal.
Teacher:	yes. If you say it in the diagram?
Student:	AR and
Teacher:	yes, AR and
Student:	QP are
Teacher:	Yes
Student:	the same.

By having the student locate the mathematical argument about the equality of sides in the diagram, the connection between a general rule and its visual instantiation is created.

The exceptional implementation of one HSR task

It is especially the case of one HSR task, asking students to construct a graph from a table of numbers, which is of interest because a closer examination may suggest how its implementation produced a mean length of connected discourse otherwise associated with LSR tasks (M (task of interest) = 11.3, M (HSR) = 4.7, M (LSR) = 8.9). How was this HSR task transformed into a LSR task, implemented at a quality higher than expected? A first look at a comparative application task also implemented during this lesson, which did not involve the activity of representing, reveals a mean of 8 connected statements. Although the discursive nature of instructional talk is still evident, a tribute to the teacher's instructional style, the HSR task was associated with talk of greater connectedness. The task statement "The elongation of the spring under force is now to be examined. A. Determine the values for the elongation of the spring as a function and enter them in a table. B. Draw- draw a diagram and indicate its coordinate assignment," intends students to follow a given procedure in order to construct a graph. An examination of discourse reveals, however, that much of the public talk occurs even before students start working on the task. This is because the teacher discovers that the students do not understand the difference between length of a spring and its elongation. By comparison to a previously constructed graph of the relation between length of the spring and force, the teacher manages to introduce the concept of elongation. It is disputable whether this explanatory talk is part of task implementation or whether it is a follow-up on the task statement. It may be noted, however, that none of the other cases analyzed involved this type of clarifying talk before the task was actually worked on by students.

Analysis of discourse occurring after students' seatwork it finished shows that the teacher not only reiterates the numerical answers for the spring's elongation depending on force but goes beyond the task statement by asking a question connecting the graph to a functional equation. That is, the teacher's question "How does Y come about?" which follows a student's attempt to formulate an equation with "Y equals…" stipulates a discussion of several turns during which students and teacher co-construct the correct equation for the graph. By attempting a connection between symbol systems, the teacher thus turns the visual result of a HSR task into the basis for further mathematical reasoning. What is missing from the conversation, however, is the use of this visual representation as a tool to anchor students' mathematical conjectures. An excerpt from the transcript illustrates that students seem to try several guesses at the correct relationship between X and Y—guesses which may have been turned into informed reasoning had the teacher pointed out on the graph how to read off the value of the slope.

Teacher:	Okay and who knows this time how it would generally work when I say how much the elongation is a X centimeters where I did not stipulate anything specifically?
Student:	Y equals
Teacher:	now you are so well trained regarding the Y here. Christian. How does this Y come about? (..) zero…ten twenty thirty
Students:	plus ten
Student:	ten plus ten equals X
Teacher:	ten plus ten? That that can't that can't be calculated that way.
Student:	zero plus ten
Teacher:	pardon? How did you correct yourself?

Student:	ten times ten...X
Student:	zero plus ten times X.
Teacher:	well I write the X first. Okay and now I enter the X. I need to get form X equals one to
Student:	ten times X.
Student:	ten times X.
Student:	ten times X.
Teacher:	ten times X. Exactly.

It thus seems that the critical element of this task implementation is the teacher's follow-up question intending students to connect symbolic systems. Nevertheless, an opportunity to focus students' attention on the crucial element of discussion—the visualization of the slope in the graphical representation—was left unexploited.

Discussion

Summarizing these results it is evident that during introductory phases of 60 mathematics lessons, representations were rarely used as a basis for student activity. Only 2.8% of introductory tasks intended students to use representations in connection to other symbol systems, even though these tasks offer a greater potential for high-level student activities than other tasks. For a comparison of ten representation tasks of similar cognitive demands, task implementations differed with respect to the structure and the content of instructional discourse. For HSR tasks, i.e., task statements focusing on one specific element or method with respect to the use of representations, a lower percentage of instructional discourse occurred in connected sequences, with sequences of significantly shorter average length than discourse associated with LSR tasks. The content of instructional discourse showed a procedural focus, with rare attempts of teachers to connect solution methods through explanatory talk. The role of the respective representation during instructional discourse was one of an end in itself.

Possibly constrained by the limiting task statement, representations were not employed to their full potential, even in cases where students' understanding of a topic discussed could have been furthered. In contrast, in LSR task, leaving open the way in which a representation may be employed during task completion, instructional discourse showed a higher degree of connected sequences, with altogether a significantly higher percentage of interconnected public talk than in HSR tasks. The content of instructional discourse more frequently involved the statement of connections between solution methods, symbol systems, or mathematical concepts. Importantly, these mathematical conjectures were not merely presented by the teacher but collaboratively constructed in student-teacher-discourse. In many instances, the visual representation served as a basis for shared reasoning, anchoring mathematical arguments. The tool-like use of a representation, emergent during accomplishment of an overall mathematical goal, apparently sustained high-level classroom discussion, while the use of a representation as an end in itself narrowed student activity to a procedural focus.

Findings of the cases analyzed suggest that the two indices for quality of instructional discourse--structure of discourse, with regard to its extent of interconnectedness, and content of discourse, with regard to its treatment of mathematical procedures and explanation--do not occur independently of each other. Based on assumptions of a socio-constructivist point of view, this interdependence is expected (see also Hiebert & Wearne, 1993). If a topic is negotiated by several students and teacher collaboratively, there is more room for the development of sophisticated argumentation as the need to explain one's reasoning to others arises frequently. Surely, similar content could have been presented by a teacher in one long explanatory statement; however, whether this would move students beyond their existing conceptualizations of a problem is questionable. Especially if the task statement was one of limited types of solution procedures, broader connections will be removed from students' constrained perception of the problem situation. This initial perception will both limit the occurrence of spontaneously constructed connections by students and their ability to relate to teacher-presented explanatory talk. Different types of task statements may thus afford a more or less tight relation between structure and content of spontaneously emerging instructional discourse.

Even if a task statement may limit students' initial conceptualizations of a problem, the use of representations during implementation bears the potential for interconnected discourse of sophisticated mathematical content. Analyses presented in this paper suggest that the implementation of a HSR task could have been turned into one of higher quality, had the representation been employed more appropriately during discussion. In cases of a limiting task statement, it thus is especially important that the teacher view and use the representation as a reasoning tool, enabling students to make connections between mathematical concepts. This will not only further discourse during implementation but also more generally students' perception of the function of representations. That is, by a teacher's modeling of the use of representations during the discussion of task solutions, students may move beyond a perception of representations as objects of closed symbol systems and come to regard them as means for the conceptualization and communication of mathematical concepts.

References

Baumert, J., Bos, W. ,& Lehmann, R. (2000). TIMSS/III. Dritte Internationale Mathematik- und Naturwissenschaftsstudie--Mathematische und naturwissenschaftliche Bildung am Ende der Schullaufbahn, Band 1: Mathematische und naturwissenschaftliche Grundbildung am Ende der Pflichtschulzeit. Opladen: Leske und Budrich.

Bloom, B., Engelhart, Furst, Hill, & Krathwoh (1956). Taxonomy of education objectives. Handbook I: The cognitive domain. New York: David McKay Co.

Cobb, P. (1995). Cultural tools and mathematical learning: A case study. Journal of Research in Mathematics Education, 26(4), 362-385.

Cobb, P., Gravemeijer, K., Yackel, E., McClain, K., & Whitenack, J. (1997). Mathematizing and symbolizing: The emergence of chains of signification in one first-grade classroom. In D. Kirshner & J. Whitson (Eds.), Situated cognition: Social, semiotic, and psychological perspectives. Mahwah, NJ: Lawrence Erlbaum Associates.

Doyle, W. (1983). Academic work. Review of Educational Research, 53, 159-199.

Dreyfus, T. (1991). On the status of visual reasoning in mathematics and mathematics education. Proceedings of the 15th international conference for the psychology of mathematics education. Genova: University of Genova.

Henningsen, M. & Stein, M.K. (1997). Mathematical tasks and student cognition: Classroom-based factors that support and inhibit high-level mathematical thinking and reasoning. Journal for Research in Mathematics Education, 28(5), 524-549.

Hiebert, J. & Wearne, D. (1993). Instructional tasks, classroom discourse, and students' learning in second-grad arithmetic. American Educational Research Journal, 30(2), 393-425.

Hiebert, J., Carpenter, T., Fennema, E., Fuson, K., Human, P., Murray, H., Olivier, A., & Wearne, D. (1996). Problem solving as a basis for reform in curriculum and instruction: The case of mathematics. Educational Researcher, 25(4), 12-21.

Kaput, J. (1989). Linking representations in the symbol systems of algebra. In S. Wagner and C. Kieran (Eds.), Research issues in the learning and teaching of algebra, Vol. 4. (pp.167-194). Hillsdale, NJ: Lawrence Erlbaum Associates.

Knoll, S. (2001). Aufgabenklassifikation in Einführungsphasen [Task Analysis of Introductory Phases]. Manuscript in preparation. Berlin: Max Planck Institute.

McGinn, M. & Roth, W.M. (1999). Preparing students for competent scientific practice: Implications of recent research in science and technology studies. Educational Researcher, 28(3), 14-24.

National Council of Teachers of Mathematics (1989). Curriculum and evaluation standards for school mathematics. Reston, VA: Author.

Renkl, A. (1991). Die Bedeutung der Aufgaben- und Rückmeldungsgestaltung für die Leistungsentwicklung im Fach Mathematik. Dissertation: Universität Heidelberg.

Roth, W.M. & McGinn, M. (1995). Differential participation during science conversations: The interaction of display artefacts, social configuration, and physical arrangements. In D.C. Edelson (Ed.), Proceedings of ICLD 96 (pp. 300-307).

Roth, W.M. & McGinn, M. (1997). Graphing: Cognitive ability or practice? Science Education, 81, 91-106.

Roth, W.M. & McGinn, M. (1998). Inscriptions: Toward a theory of representing as social practice. Review of Educational Research, 68(1), 35-59.

Schümer, G. (1997). TIMSS: Videotape Classroom Study. Discourse Coding Manual. Unpublished Manuscript. Berlin: Max Planck Institute for Human Development.

Sfard, A. (2000). Symbolizing mathematical reality into being—Or how mathematical discourse and mathematical objects create each other. In P. Cobb, E. Yackel, and K. McClain (Eds.), Symbolizing and commincating in mathematics classrooms (pp.37-97). Mahwah, NJ: Lawrence Erlbaum Associates

Stein, M-K., Grover, B., & Henningsen, M. (1996). Building student capacity for mathematical things and reasoning: An analysis of mathematical tasks used in reform classrooms. American Educational Research Journal, 33(2), 455-488.

Stigler, J., Gonzales, P., Kawanaka, T., Knoll, S., & Serrano, A. (1996). The TIMSS videotape classroom study: Methods and preliminary findings. Report prepared for the National Center for Education Statistics, U.S. Department of Education. Los Angeles: University of California at L.A.

Appendix A.

High Structure Representation Tasks:

Um for the first one...um has to do with the relationship between X and Y right? Check your own tables yourself, take the points first and then draw the graph. In the second one use the slope and intercept and write it please.

And what sort of equation uh straight line equation or basically linear function equation will be made? So let's try to solve for that please. We should use the same thinking as before of up until now the draw- drawn oh of when we have drawn the graph okay? The equation itself is...like I have said before is...always in this form. It will be in this form. While checking the amounts for A and B okay? By yourself. ..the explanation we will do later so try to do it by yourself.

The elongation of the spring under force is now to be examined. A. Determine the values for the elongation of the spring as a function and enter them in a table. B. draw- draw a diagram and indicate its coordinate assignment.

Use a graphing utility to graph each of the following quadratic equations. Sketch the graph on the axis provided. State the number of X intercepts and evaluate the discriminant.

As written here um it's that there are two line segments AB CD. Those are intersected at point O but if there are conditions that say AO and CO are equal and DO and BO are equal then it's that the triangle we call AOD and the triangle we call COB become congruent. Okay? Well okay? Please draw the figures and try doing it. Draw the figures and...

Low Structure Representation Tasks:

A plus B. Please put a plus sign there between A and B. On the one side above and on that side also. And of course you could also place parentheses. And now try using guidelines to construct this square so that you can recognize the parts of the formula again.

So I prepared the next diagram and I'll leave it up to your imaginations.

In the time leftover um why? Okay? Why the sum total of PQ and PR are the same as AB. Please find the reasons for it. Okay? Do you understand? Okay? Then if you've found it and under- understood it, make it into words, you understand. Even if you can't make it into words, okay? If it's understandable and be understood, it's okay. Just the important parts.

You are going to develop a plan and you are going to come up with an estimate based on the mathematics of...how fast this plane is flying.

Okay. Um. It has been one month since Ichiro's mother has entered the hospital. Um. He has decided to give a prayer with his smaller brother at a local temple every morning so that she will be well soon. Um. There are eighteen ten yen coins in Ichiro's wallet and just twenty-two five yen coins in his smaller brother's wallet. They have decided every time to take one coin from each of them and put them in the offertory box and continue the prayer up until either wallet becomes empty. One day after they were done with their prayer when they looked into each other's wallets the smaller brother's amount of money was bigger than Ichiro's. How many day has it been since they started the praying? That's the problem.

Now I think there are points that are a little too hard to understand with just the sentences so well I think I would like to look at a figure and check it.

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